國立臺灣大學 National Taiwan University

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Lab On a Chip:

Microfluidics (I)

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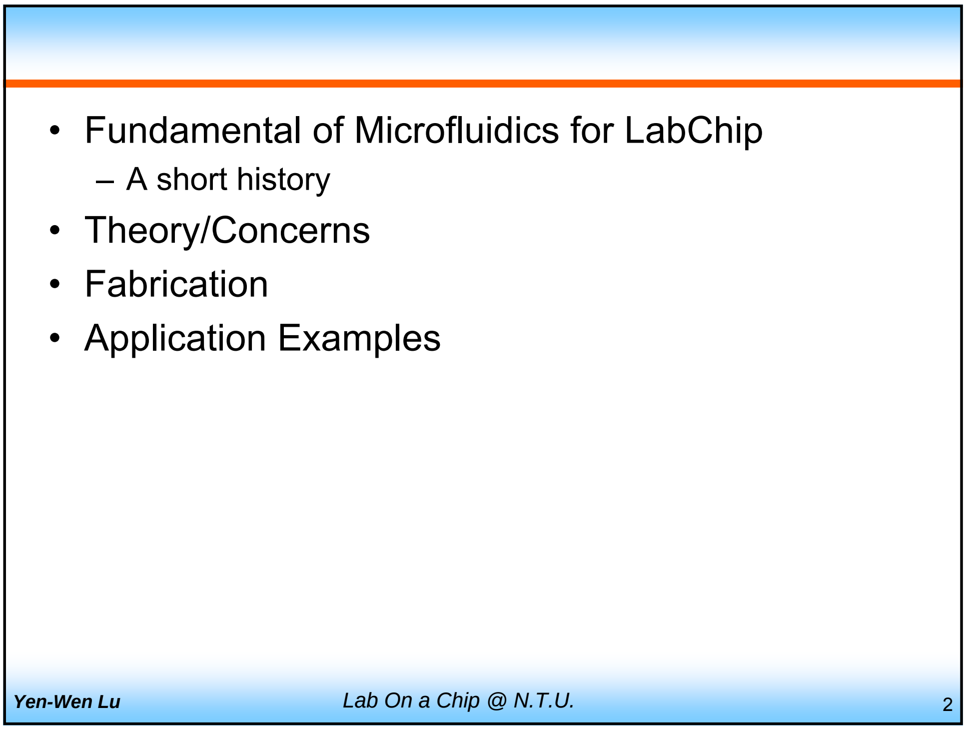


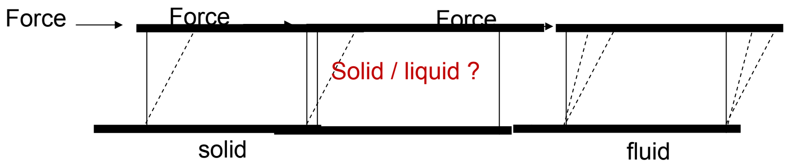
- Fundamental of Microfluidics for LabChip
  - A short history
- Theory/Concerns
- Fabrication
- Application Examples

Preface

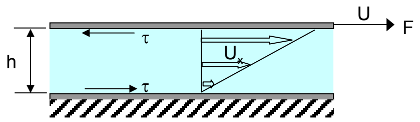
- Fundamental of Microfluidics for LabChip
- Microfluidics vs. Microelectronics
- Fluid Mechanics at microscale
  - Classical fluid mechanics
  - Microfluidic: classical + electrostatics, thermodynamics, statistical mechanics, elasticity …etc
- Best known in microfluidics:
  - Fluidic systems reach length scales where the fundamental fluid physics changes dramatically
    - Mass transport in microfluidics is dominated by viscous dissipation
    - Inertial effects are generally negligible.
    - Microfluidic physics is quite rich.

Fluids Basics

- Solids vs Fluids



- Newtonian Flow



Couette flow

- Flowing fluids can be characterized by the properties of both the ***fluids*** and the ***flow***
  - **Kinetic properties**: linear and angular velocity, vorticity, acceleration, strain rate, ..
  - **Transport properties**: viscosity, thermal conductivity, diffusivity, …
  - **Thermodynamic properties**: pressure, temperature, density, …
  - **Miscellaneous properties**: surface tension, vapor pressure, ..

Modeling of a Fluid Flow

- **As a continuum in which properties are defined to be continuously throughout space. (continuum approach)**
- **As a collection of individual interacting molecules (molecular approach)**

As the length scale of a system decreases in size, the question of whether to treat the fluid as a collection of molecules or as a continuum acquires critical significance.

→ Using a continuum approach in the situation where a molecular approach is necessary would certainly produce incorrect results.

→ Knudsen number (1900)

Channel Classification - Issues

- Single-phase gas flow

Knudsen number $Kn = \frac{\lambda}{D_h}$

λ : mean free path $\lambda = \frac{\mu\sqrt{\pi}}{\rho\sqrt{2RT}}$

Range of Kn	Type of Flow
0.001≧Kn	Continuum flow, no rarefaction effects
0.1≧Kn≧0.001	Slip flow, rarefaction effects modeled with wall slip
10≧Kn≧0.1	Transition flow, statistical analysis
Kn≧10	Free molecular flow, motion of individual molecules modeled and then treated statistically

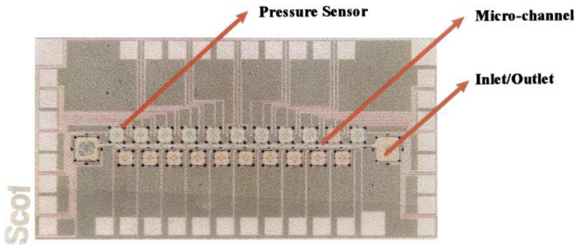
Mean Free Path for Various Gas @ 1atm, 300K

Gas	Mean Free Path, μm
Air	0.068
Helium	0.194
Hydrogen	0.125
Nitrogen	0.066

Channel Dimension in Different Flow Regions @ 1atm, 300K

Gas	Continuum Flow	Slip Flow	Transition Flow	Free Molecular Flow
Air	> 67 μm	0.67 ~67 μm	0.0067~0.67 μm	< 0.0067 μm
Hydrogen	> 194 μm	1.94 ~194 μm	0.0194~1.94 μm	< 0.0194 μm
Helium	> 123 μm	1.23 ~123 μm	0.0123~1.23 μm	<0.0123 μm

Pressure Distribution



CM, Ho. et al, MICRO-ELECTRO-MECHANICALSYSTEMS (MEMS) AND FLUID FLOWS, AnnualReviewofFluidMechanics 1998

Intermolecular Forces

- The behavior of all states of matter-solids, liquids, and gasses –as well as the interaction among the different states depends on the forces between the molecules that comprise the matter.
- An accurate model of the interaction of two simple, nonionized, nonreacting molecules is given by the Lennard-Jones potential:
  - r: the distance separating the molecules i and j
  - c_{ij} and d_{ij} are parameters particular to the pair of interacting molecules
  - ε is a characteristic energy scale
  - σ: characteristic distance of the two molecules

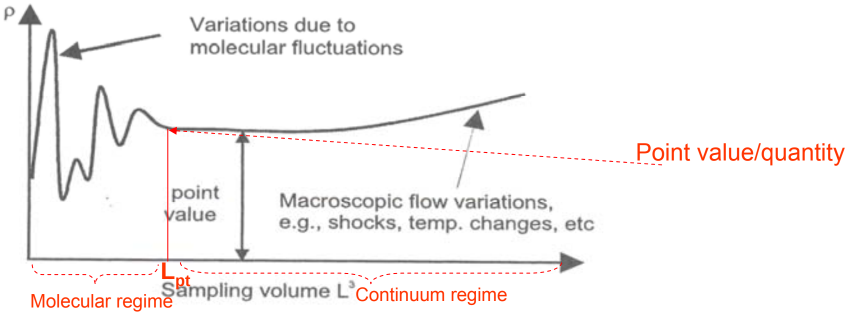
Pairwise repulsion that exists between two molecules

Attractive potential due to the van der Waals force between any two molecules

van der Waals
- Dipole/dipole interactions
- Induced dipole interactions
- Fundamental electrodynamic interactions

Continuum Assumption

- All quantities of interest such has density, velocity, and pressure are assumed to be defined everywhere and to vary continuously from point to point within a flow
- If the molecules of a liquid are closed packed relative to the length scale of the flow, the continuum assumption is probably valid
- Average particle density vs sampling volume

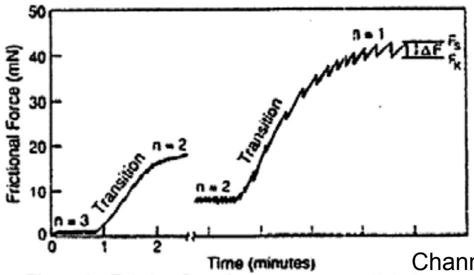


When the number of molecules in the sampling volume is small, the calculated density would vary rapidly
Eg. $L_{gas,pt}=10^{-6}$ m (based on 1% reasonable statistics, 10^4 molecules)

Molecular Effects on Viscosity

- When the length scale of the flow field is downscaled to "the size of the molecules", the continuum assumption may not be valid.
- The viscosity shows a departure from its typical value.
- When channel size is below the 10 molecule diameters, the fluid loses its liquid-like behavior and assumes solid-like characteristics
- Structure of liquid in a molecule-size channel changes from a random order to a discrete number of ordered layers; viscosity can be as high as 10^5 of the apparent viscosity

- Stepwise increase in friction

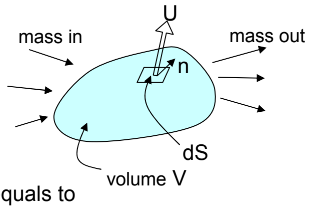


Channel size decreases

Figure 1. *Friction force between two parallel plates*

Conservation of Mass

•Total mass *m* inside the volume



•The time rate of change of mass inside the control volume equals to the negative of the outflow of mass

•Since the control volume is fixed in space, we can take the time derivative in the integral

Time Rate of Changes of Momentum

Momentum ***p***

With the Reynolds’ Transport Theorem and considering a fixed volume

The Navier-Stoke Equation

- Newton’s law of motion
- Three components of the total force:
  1. The net pressure acting normal to the bounding surface
  2. The net shear force acting tangential to the surface
  3. Body force, such as gravity

Navier-Stoke Equation

- Incompressible Flow : the time rate of change of density is negligible, and the continuity equation for mass conservation becomes: $\nabla \cdot U = 0$

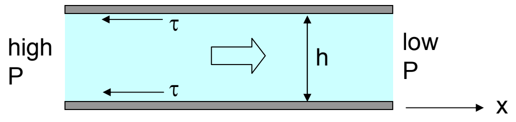
- Reynolds Number : Re is a ratio that measures the relative importance of inertial forces to viscous forces.

Stoke Flow

when the Reynolds number is much less than unity, a typical situation in MEMS devices.

Poiseuille Flow

A pressure-driven flow with uniform pressure gradient



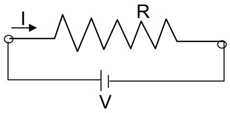
- Shear stress on the wall

- Relation between pressure drop and flow

$$\Delta P = \frac{12\mu L}{Wh^3}Q$$

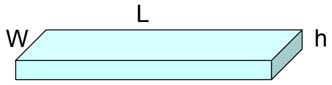
, when W>>h **(Poiseuille's Law)**

- In the fluid domain, using the lumped element model



$$R_{pois} = \frac{12\mu L}{Wh^3}$$

e.g.

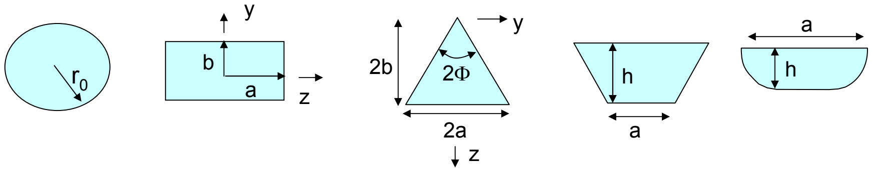


- **(W,h,L)= (100 μm, 30μm, 3cm), Q= 1 μl/min**
- **(W,h,L)= (1 μm, 0.3μm, 3cm), Q= 1 μl/min**

$\mu_{water,\ 20\ C} = 11.3 \cdot 10^{-4}$ (Pa·s)

Pressure-driven pumping becomes very difficult !!

Parallel Flow in different-geometry channels



- For circular cross section

Flow velocity: $u_1 = \frac{-dp/dx}{4\mu}(r_0^2 - r^2)$ Flow rate: $\dot{Q} = \frac{\pi r_0^4}{8\mu}(-\frac{dp}{dx})$

Pressure drop: $\Delta p = \frac{8\mu \dot{Q} L}{\pi r_0^4}$

- For rectangular cross section

$$u_1 = \frac{16a^2}{\mu\pi^3}(-\frac{dp}{dx})\sum_{i=1,3,5..}^{\&}\left[1 - \frac{\cosh(i\pi z/2a)}{\cosh(i\pi b/2a)}\right]\frac{\cos(i\pi y/2a)}{i^3}$$

Triangular (isosceles) cross section

$$u_1(y,z) = \frac{1}{\mu}(-\frac{dp}{dx})\frac{y^2 - z^2\tan^2\phi}{1-\tan^2\phi}\left[(\frac{z}{2b})^{B-2} - 1\right]$$

- Hydraulic diameter

$$D_h = \frac{\text{4 x Cross Section Area}}{\text{Wetted Perimeter}} = \frac{4A}{P_{wet}}$$

Flow Actuation in Microsystems

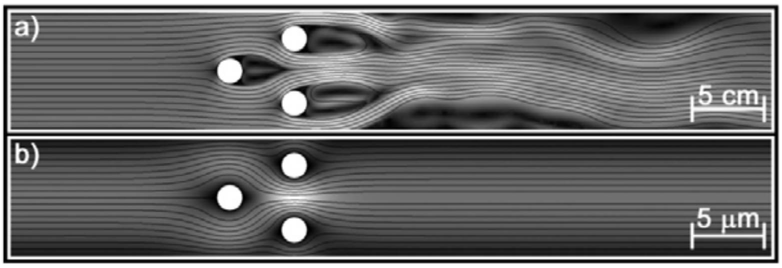
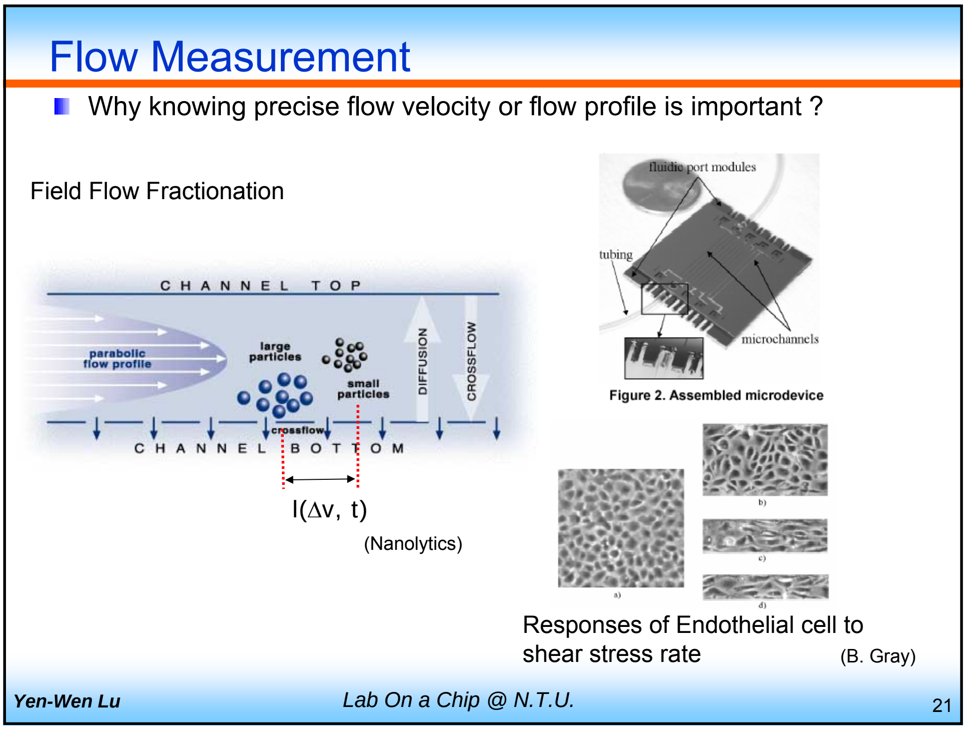


Figure 1.2 Two different fluid flows around passive obstacles, where the design is the same but only scaled. The macro regime (panel a), diameter 0.1m and fluid velocity 0.1m/s, $Re = 10^4$) shows a turbulent flow with vortices after the obstacles. In the micro regime (panel b), diameter 0.1mm and fluid velocity 100µm/s, $Re = 0.01$) no visible disturbances are introduced after the obstacles because of the laminar flow conditions.

PhD Dissertation by Roy Josephus Stephanus Derks 2010

Difficulties in measuring accurate flow velocity

- Variations in bulk flow rates
- Variations in channel geometry
- In- situ flow measurement



Flow Measurement

- Why knowing precise flow velocity or flow profile is important ?

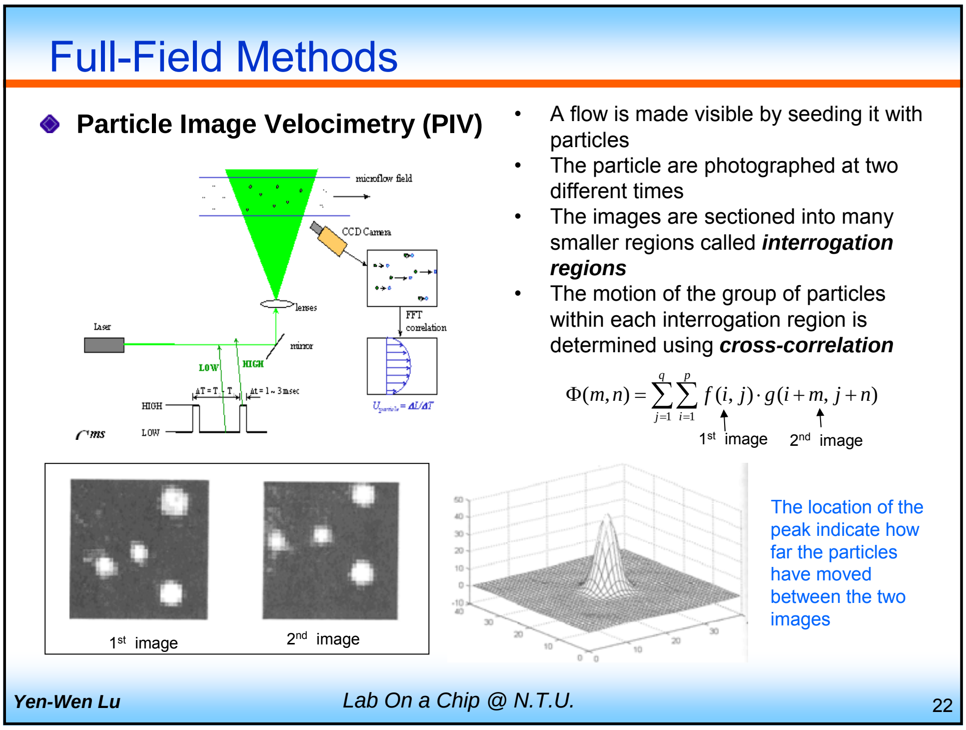
Field Flow Fractionation

(Nanolytics)

Figure 2. Assembled microdevice

Responses of Endothelial cell to shear stress rate

(B. Gray)



Full-Field Methods

Particle Image Velocimetry (PIV)

- A flow is made visible by seeding it with particles
- The particle are photographed at two different times
- The images are sectioned into many smaller regions called ***interrogation regions***
- The motion of the group of particles within each interrogation region is determined using ***cross-correlation***

$$\Phi(m,n)=\sum_{j=1}^{q}\sum_{i=1}^{p} \underset{\uparrow}{f(i,j)}\cdot \underset{\uparrow}{g(i+m,j+n)}$$

1st image 2nd image

1st image 2nd image

The location of the peak indicate how far the particles have moved between the two images

PIV vs μPIV

- What are the major difficulty moving macroscopic PIV to microscopic PIV?
    - Seeding particles
    - Brownian motions
    - Flow illumination

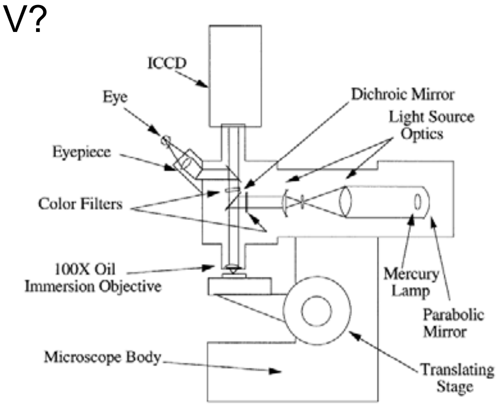


Fig. 1. Schematic of the micro-PIV recording system using: (1) an epifluorescent microscope, (2) a high numerical aperture lens, (3) a continuous-illumination mercury arc lamp, and (4) an intensified/cooled CCD camera

Santiago, 1998

Take Home Message (Last Week)

- Classical fluid mechanism
    - Assumption?
- Inertial irrelevance?
    - Re
- When molecules matters?
    - Kn

Homework 2: G. Whitesides

- Engineering Cell Shape and Function, Science, 1994
- Chaotic Mixer for Microchannels, Science 2002
- Fabrication of a modular tissue construct in a microfluidic chip, Lab Chip, 2008
- Paper-based capacitive touch pad, Advanced Material, 2012

Homework 3:

- Micro-PIV and microfluidics
  - Prof. CD Meinhart @ UCSB (551 times since 1999)
  - Prof. JG Santiago @ Stanford (752 times since 1998)
  - Prof. Ronald J. Adrian @ ASU (2283 times since 1991)